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Research Paper

Applications of weather derivatives in the energy market

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ABSTRACT

This paper analyzes various types of weather-related risks in different industries. Due to the existence and popularity of the established weather insurance market, we also discuss the differences between weather derivatives and insurance, and the advantages of using weather derivatives instead of insurance in weather risk management. First, we discuss the applications of temperature futures by providing a static, simple strategy to hedge volume risks in practice. Then, by building up a system of models for energy and temperature, we propose a dynamic hedging strategy in order to hedge energy futures using temperature futures.

Keywords: weather derivatives; mean-reverting process; energy markets; dynamic hedging; crude oil futures.

1 INTRODUCTION

Weather, and especially temperature, risks have a significant impact on the operational and financial decisions and revenues of various industries, such as energy producers, distributors and retailers. Due to the notably increasing occurrences of extreme weather and temperature volatility at high latitudes, weather is increasingly

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TABLE 1 Industries, weather exposure and types of risk.

Industry	Weather type	Risk
Energy consumer	Temperature	Excessive or reduced demand
Energy industry	Temperature, wind, solar irradiance, precipitation	Excessive or reduced supply
Agriculture	Temperature, precipitation	Crop yield, handling, storage, pests
Retailing	Temperature, precipitation	Reduced demand
Travel	Temperature, precipitation, snowfall	Cancelations or lower revenue
Transportation	Temperature, precipitation, wind, frost day, snowfall	Delays, cancelations, higher operational costs, lower revenues
Government	Temperature, precipitation, snowfall	Higher budget costs
Construction	Temperature, precipitation, snowfall	Delays, higher budget costs

affecting supply and demand in businesses. In these circumstances, hedging strategies and weather derivatives that can be deployed to transfer risk exposures to willing counterparties become increasingly valuable and useful. In applying weather derivatives to other commodities markets, the primary aim of this paper is to consider the cross-hedging strategy for other commodity derivatives using weather futures contracts.

1.1 Weather risks

Weather conditions affect our food, clothing, shelter and transportation – almost all the basic necessities of life. In recent years, several studies (see, for example, Dutton 2002) have estimated that more than one-third of private economic activity in the United States is exposed to weather risks, revealing that weather is a major factor that affects almost all economic activity. In Table 1, we list some of the industries that are exposed to weather risk and the particular risk(s) that they are facing. From Table 1, we can conclude that weather affects economies worldwide and has an impact on either revenues or costs, or both.

In the risk management arena, two types of weather risk can be defined (see Leggio 2007). A weather risk that is referred to as a high-probability, limited-loss weather event is categorized as a "noncatastrophic weather risk", whereas a low-probability, huge-loss weather catastrophe is defined as a "catastrophic weather risk". The impact of catastrophic weather risks has been recognized, acknowledged and managed directly by insurance contracts. Noncatastrophic weather relates to an extreme deviation from the usual weather, such as warmer winters or cooler summers. Hereafter, we are concerned with the hedging of noncatastrophic weather risks.

Based on the description in Cogen (1998), noncatastrophic weather risk can be defined as the uncertainty in future cashflows as a result of seasonal variations in the weather. This type of risk differs from other financial risks in the following aspects:

- weather risk is a "volume" risk that affects quantity but not price;
- weather risk is a highly localized risk;
- weather risk has low correlation with other financial risks;
- there is no physical market for weather indexes, and weather is a purely exogenous risk that is beyond people's control.

From the description above, we know that, as a source of risk, weather is specific because it primarily affects the quantity of supply or demand for a certain product. This is the main reason that the weather risk we are talking about here is a volumetric risk rather than a significant price risk.

1.2 Weather derivatives and insurance

In the area of financial risk management, insurance and derivatives are two major financial risk management tools. In a sense, weather derivatives are a combination of insurance and capital markets. However, weather derivatives provide benefits that weather insurance does not. First, to make a claim on an insurance contract, the holder must provide evidence of the damage and assess the losses directly caused by a specific weather event. Unlike insurance, a weather derivative offers a payoff with a value based solely on the output of the weather index measured and provided by an objective third-party agency. Second, hedging with weather derivatives has fewer contracting costs, since there is less moral hazard and there are fewer adverse counterparties involved in such trading. Third, weather derivatives are more convenient than insurance to protect against high-probability events with limited losses. Hence, in general, weather derivatives offer various benefits over alternative weather risk management tools, since they can

- transfer weather-related risks to a party that can manage them more efficiently,
- provide compensation for losses that occur,
- offer a payoff simply based on weather index value (a field inspection is not needed),
- eliminate the insurable interest in the subject of insurance,
- be hedged relatively easily, since weather risk is primarily a volume risk.

With these benefits, it is natural for industries to use weather derivatives to stabilize their revenues, cover over-budget costs and reimburse losses. Moreover, since the underlying indexes of weather derivatives have limited correlation with other financial indexes (see Cao *et al* 2004; Platen and West 2004), they can also be used as an alternative asset class to diversify investment portfolios.

2 HEDGING WITH TEMPERATURE FUTURES

In this section, as an application of weather derivatives, we use futures contracts written on temperature to demonstrate the hedging strategies for commodities. No payment is required to enter a futures contract position since the probability of a weather event being lower or higher than the threshold is the same for both sides; either side has the same chance of receiving a payoff from the counterparty.

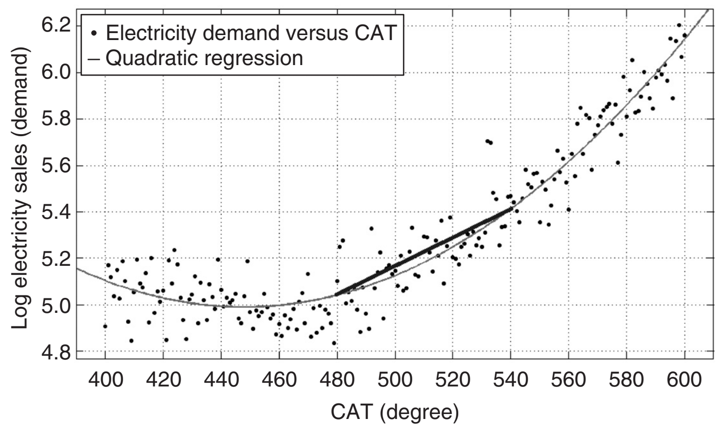
We introduce two types of hedging strategies using temperature futures in the following sections. The first strategy is a static hedging that mainly focuses on mitigating the volume risk of commodity sales using temperature futures. The other strategy we consider is the dynamic hedging strategy of commodity futures using temperature futures. Without loss of generality, we choose to hedge the energy market using temperature futures in this section.

2.1 Static hedging for volume risks

As we discussed in the introduction, many industries, especially energy utilities, are exposed to weather risks. From the heating oil spot price example in Section 2.2, we can see that there is a limited price risk related to weather seasonality. Even if this is not the case for other commodities, people can still use other hedging instruments, such as more suitable corresponding commodities futures, to hedge the price risk. Hence, the weather risk we want to hedge here is mainly the volume risk faced by the energy sector.

A typical example would be an electricity retailer in Canada experiencing a low demand on electricity (for air conditioners) during an unusually cool July. With the cumulative average temperature (CAT) future available with price C$540 in April, the

FIGURE 1 Illustration of log electricity sales (demands) versus CAT index in July.



firm could enter a short position on a CAT futures contract with the month of July being the measurement period. If the finally realized CAT index is C$480, which means July eventually becomes cool,[1] the seller of the CAT future will receive a reimbursement of C$60 $\times$ 20 $=$ C$1200 for the low sell volume of electricity in July.[2] To construct a static hedging strategy for this electricity retailer firm, suppose the weather-dependent electricity sales (demands) have the empirical illustration shown in Figure 1. Let P be the marginal profit of achieving an additional log demand of 1%, and let E be the estimated marginal effect of a 1°C CAT on log demand, starting from the realized CAT index. Based on Figure 1, E is the slope of the solid black line, which connects the realized CAT (C$480) and the market future price of the CAT (C$540). Then, if we hold a number, h, of CAT futures with S as the tick value of the CAT future, the exposure to the demand fluctuations caused by temperature is approximately $PE(\text{CAT} - F_{\text{CAT}})$, and the benefit of holding h CAT futures is $hS(\text{CAT} - F_{\text{CAT}})$. Hence, our hedging strategy under no transaction cost is to sell $h^\star$ CAT futures, such that $PE = h^\star S$ (ie, the optimal static initial hedge ratio $h^\star = PE/S$).

2.2 Dynamic hedging with temperature futures

Next, we will focus on the dynamic hedging strategy of energy futures using temperature futures. In the spirit of Bradie and Jain (2008), we consider a portfolio at time

[1] Since the average temperature for each day in July was only about 15°C in this case.
[2] The Chicago Mercantile Exchange (CME) tick size for the Canadian CAT index is C$20 per index point.

t containing one unit of energy (eg, heating oil) futures F_{E} and β_t units of weather futures F_{W},[3] both with maturity (delivery) at time T.[4] Assume the portfolio has the value $\Pi(t)$ at time t and a constant risk-free interest rate r. Then,

$$\Pi(t) = \mathrm{e}^{-r(T-t)}[F_{\mathrm{E}}(t) + \beta_t F_{\mathrm{W}}(t)]. \tag{2.1}$$

The portfolio is self-financing, so the change in this portfolio over a small amount of time $\mathrm{d}t$ is given by

$$\mathrm{d}\Pi(t) = r\Pi(t)\,\mathrm{d}t + \mathrm{e}^{-r(T-t)}[\mathrm{d}F_{\mathrm{E}}(t) + \beta_t\,\mathrm{d}F_{\mathrm{W}}(t)]. \tag{2.2}$$

Hence, in order to dynamically hedge the energy future F_{E} with maturity T, the stochastic component of the portfolio vanishes and the hedge ratio β_t can be defined as

$$\beta_t = -\frac{\mathrm{d}F_{\mathrm{E}}(t)}{\mathrm{d}F_{\mathrm{W}}(t)}, \tag{2.3}$$

with an assumption that $\mathrm{d}F_{\mathrm{W}}(t) \neq 0$. Therefore, from (2.3), to hedge an energy future, we are required to hold β_t units of temperature future at time t. Then, we need to specify two models for energy and temperature futures so that we can get the explicit dynamics of energy and temperature futures, and hence a closed form of the hedge ratio β_t. For the purpose of futures pricing, these models will be built on the underlyings of futures: namely, the energy spot price and the daily average temperature.

Note that another type of hedging ratio is called the optimal hedge ratio (see, for example, Hatemi-J and Roca 2006; Yeh 2008), which takes the form

$$\beta_t = -\frac{\operatorname{cov}(\mathrm{d}F_{\mathrm{E}}(t),\, \mathrm{d}F_{\mathrm{W}}(t))}{\operatorname{var}(\mathrm{d}F_{\mathrm{W}}(t))}, \tag{2.4}$$

where cov is the covariance and var is the variance. If we are clear about the dynamics of the energy and temperature futures, it is also possible to apply this dynamic hedging strategy.

3 ENERGY AND TEMPERATURE MODELS

3.1 Energy model

As a classical commodity model, and the most widely used for pricing energy derivatives, we consider Schwartz's celebrated model; for simplicity, we use the one-factor

[3] β_t is the hedge ratio for weather future F_{W}.
[4] We will discuss the setting of a temperature future measurement period corresponding to the energy future delivery time later on.

model in Schwartz (1997), ie, the natural logarithm of the spot price of the energy commodity is assumed to follow a mean-reverting process driven by Brownian motion.

As usual, we assume trading in the market takes place in the time interval $[0, \mathcal{T}]$ with a finite time horizon $\mathcal{T} < \infty$. Suppose, given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, the energy spot price $S(t)$ at day $t \leqslant \mathcal{T}$ follows the stochastic process

$$\mathrm{d}S(t) = \kappa(\mu - \ln(S(t)))S(t)\,\mathrm{d}t + \sigma S(t)\,\mathrm{d}W(t), \tag{3.1}$$

where κ is the mean-reversion speed, μ is the long-run mean to which the log-spot price reverts, σ, as usual, is a measurement of the volatility and $W(t)$ is the standard Brownian motion. This model is widely studied and has closed-form solutions. Moreover, we only need to estimate three parameters in this model, which makes it convenient to calibrate as well.

If we define $X(t) = \ln(S(t))$, which is the log of the spot price, and apply Ito's lemma, we get the $X(t)$ dynamics of the stochastic differential equation

$$\mathrm{d}X(t) = \kappa\left(\mu - \frac{\sigma^2}{2\kappa} - X(t)\right)\mathrm{d}t + \sigma\,\mathrm{d}W(t). \tag{3.2}$$

Note that (3.2) is a standard Ornstein–Uhlenbeck process driven by Brownian motion.

Our aim is to derive the arbitrage-free and explicit dynamics for the future price $F_{\mathrm{E}}(t)$; therefore, we need to specify an equivalent martingale measure Q as the pricing measure first. Since the P-measure dynamics of the spot price $S(t)$ are driven by Brownian motion, we choose the probability measure Q using a Girsanov transformation. We assume a constant market price of risk $\theta < \infty$,[5] then define the stochastic process

$$Z^{\theta}(t) = \exp\left(\int_0^t \theta\,\mathrm{d}W(t) - \frac{1}{2}\int_0^t \theta^2\,\mathrm{d}s\right). \tag{3.3}$$

Hence, the probability measure Q^{θ} can be defined with the density process Z^{θ} over $[0, \mathcal{T}]$ as

$$Q^{\theta}(A) = \mathbb{E}[\mathbf{1}_A Z^{\theta}(\mathcal{T})], \tag{3.4}$$

where $\mathbf{1}_A$ is the indicator function over probability space A, and $\mathcal{T}$ is a fixed bounded time greater than the current time t. Based on the Girsanov transformation, the Brownian motion under measure Q^{θ} is then defined as

$$\mathrm{d}W^{\theta}(t) = \mathrm{d}W(t) - \theta\,\mathrm{d}t. \tag{3.5}$$

The dynamics of $X(t)$ under measure Q^{θ} then become

$$\mathrm{d}X(t) = \left(\theta\sigma + \kappa\left(\mu - \frac{\sigma^2}{2\kappa} - X(t)\right)\right)\mathrm{d}t + \sigma\,\mathrm{d}W^{\theta}(t). \tag{3.6}$$

[5] For illustration, we consider the market price of risk here as a bounded constant.

Further, by application of Ito's lemma, for $\tau \geqslant t$ the stochastic process $X(\tau)$ has the explicit solution under measure Q^θ defined as

$$X(\tau) = X(t)\mathrm{e}^{-\kappa(\tau-t)} + \frac{1}{\kappa}\left(\theta\sigma + \kappa\left(\mu - \frac{\sigma^2}{2\kappa}\right)\right)(1 - \mathrm{e}^{-\kappa(\tau-t)}) + \int_t^\tau \sigma \mathrm{e}^{\kappa(s-\tau)}\,\mathrm{d}W^\theta(s). \tag{3.7}$$

From the solution under measure Q^θ and Ito isometry, the log-spot price $X(\tau)$ conditioned on $X(t)$ is normally distributed under Q^θ with expectation

$$\mu_X(\tau) := \mathbb{E}^\theta[X(\tau) \mid X(t)] = X(t)\mathrm{e}^{-\kappa(\tau-t)} + \frac{1}{\kappa}\left(\theta\sigma + \kappa\left(\mu - \frac{\sigma^2}{2\kappa}\right)\right)(1 - \mathrm{e}^{-\kappa(\tau-t)}), \tag{3.8}$$

and variance

$$\sigma_X^2(\tau) := \mathrm{Var}^\theta[X(\tau) \mid X(t)] = \int_t^\tau \sigma^2 \mathrm{e}^{2\kappa(s-\tau)}\,\mathrm{d}s = \frac{\sigma^2}{2\kappa}(1 - \mathrm{e}^{-2\kappa(\tau-t)}). \tag{3.9}$$

With this result, we can move on to calculate the future price for the energy commodity.

Assuming a constant risk-free interest rate, the future price at current trading time t of an energy commodity with delivery time T, $0 \leqslant t < T$, is the expected price of the commodity at time T under the equivalent martingale measure. Since we know that

$$X(T) \mid X(t) = \ln(S(T)) \mid \ln(S(t)) \sim N(\mu_\mathrm{E}, \sigma_\mathrm{E}^2),$$

$S(T) \mid S(t)$ is then lognormally distributed. From the first-moment formula of the lognormal distribution,

$$F_\mathrm{E}(t, T) = \mathbb{E}^\theta[S(T) \mid S(t)] = \exp(\mu_X(T) + \tfrac{1}{2}\sigma_X^2(T)), \tag{3.10}$$

where $\mu_X(\tau)$ and $\sigma_X^2(\tau)$ are given in (3.8) and (3.9).

Next, we derive the dynamics of the energy future price $\mathrm{d}F_\mathrm{E}(t, T)$ under measure Q^θ. If we recall (3.6) and apply Ito's lemma, we get

$$\mathrm{d}F_E(t, T) = \left(\frac{\partial F_\mathrm{E}}{\partial t} + \frac{1}{2}\frac{\partial^2 F_\mathrm{E}}{\partial X(t)^2}\right)\mathrm{d}t + \frac{\partial F_\mathrm{E}}{\partial X(t)}\,\mathrm{d}X(t). \tag{3.11}$$

As long as the future price process in (3.10) is the conditional expectation of the spot price at maturity time T, the future price process $F_\mathrm{E}(t, T)$ is a Q^θ martingale, and all coefficients of the $\mathrm{d}t$ term vanish. In the Q^θ dynamic of $X(t)$, the only coefficient of the $\mathrm{d}W^\theta(t)$ term is σ. By calculating the partial derivative $\partial F_\mathrm{E}/\partial X(t)$ based on (3.10) and multiplying by σ, we get

$$\mathrm{d}F_\mathrm{E}(t, T) = \sigma \mathrm{e}^{-\kappa(T-t)} \exp(\mu_X(T) + \tfrac{1}{2}\sigma_X^2(T))\,\mathrm{d}W^\theta(t), \tag{3.12}$$

where $\mu_X(\tau)$ and $\sigma_X^2(\tau)$ are given in (3.8) and (3.9).

3.2 Temperature model

Because the dynamic hedging strategy we are considering here also needs the explicit weather futures dynamics, we have to consider a temperature model to derive the dynamics of temperature futures. Without loss of generality, we focus on using CAT futures to hedge the energy future as an example of weather futures. We choose the underlying temperature model for the CAT future to be the widely used Ornstein–Uhlenbeck model (Benth and Šaltytė-Benth 2005; Swishchuk and Cui 2013).

Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, suppose the daily average temperature $T(t)$ on day t with $0 \leqslant t \leqslant \mathcal{T} < \infty$ follows the stochastic process

$$\mathrm{d}T(t) = \mathrm{d}s(t) + \kappa(T(t) - s(t))\,\mathrm{d}t + \sigma(t)\,\mathrm{d}W(t), \tag{3.13}$$

where $s(t)$ is the seasonal mean level of temperature, κ is the mean-reverting speed, $\sigma(t)$ is volatility and $W(t)$ is a standard Brownian motion. This type of stochastic model was introduced and studied by Dornier and Queruel (2000) and Alaton *et al* (2002) and proven to be particularly suitable for capturing the evolution of temperature through time. Note that, to get the explicit dynamics of the CAT future price, unlike the daily average temperature model in Swishchuk and Cui (2013), which is driven by the general Lévy process, we choose the random variable in the model to be a standard Brownian motion. When we compare this Ornstein–Uhlenbeck model with the one we used to model the energy spot price in (3.1), we find that the model in (3.13) adds the long-term seasonal mean $s(t)$ term. The reason for this is that Dornier and Queruel (2000) found that if the long-term mean level $s(t)$ is a time-varying deterministic function (not a constant), then the term $\mathrm{d}s(t)$ needs to be added so that the dynamics $T(t)$ actually revert to the mean $s(t)$.

To derive the dynamic of the CAT future price, we use the real measure P-equivalent martingale measure Q^{θ} given by the Girsanov transformation.[6] Recall that $\mathrm{d}W^{\theta}(t) = \mathrm{d}W(t) - \theta\,\mathrm{d}t$ is a standard Brownian motion under measure Q^{θ}. Hence, the Q^{θ} dynamics of $T(t)$ becomes

$$\mathrm{d}T(t) = \mathrm{d}s(t) + (\theta\sigma(t) + \kappa(T(t) - s(t)))\,\mathrm{d}t + \sigma(t)\,\mathrm{d}W^{\theta}(t). \tag{3.14}$$

By application of Ito's lemma, for $\tau \geqslant t$ the stochastic process $T(\tau)$ has an explicit solution under the Q^{θ} measure given by

$$\begin{aligned} T(\tau) = s(\tau) + (T(t) - s(t))\mathrm{e}^{\kappa(\tau - t)} \\ + \int_t^{\tau} \theta\sigma(u)\mathrm{e}^{\kappa(\tau-u)}\,\mathrm{d}u + \int_t^{\tau} \sigma(u)\mathrm{e}^{\kappa(\tau-u)}\,\mathrm{d}W^{\theta}(u). \end{aligned} \tag{3.15}$$

[6] This Q^{θ}-equivalent measure has the same measure definition used in (3.4) for the energy market.

Again, we can see from (3.15) that the daily average temperature $T(\tau)$ conditional on $T(t)$ is normally distributed under a Q^θ measure with expectation

$$\mu_T(\tau) := \mathbb{E}^\theta[T(\tau) \mid T(t)] \\ = s(\tau) + (T(t) - s(t))\mathrm{e}^{\kappa(\tau-t)} + \int_t^\tau \theta\sigma(u)\mathrm{e}^{\kappa(\tau-u)}\,\mathrm{d}u, \qquad (3.16)$$

and variance

$$\sigma_T^2(\tau) := \mathrm{Var}^\theta[T(\tau) \mid T(t)] = \int_t^\tau \sigma^2(u)\mathrm{e}^{2\kappa(\tau-u)}\,\mathrm{d}u. \qquad (3.17)$$

A straightforward use of Swishchuk and Cui (2013, Theorem 1) would give us the future price $F_{\mathrm{CAT}}(t, \tau_1, \tau_2)$ at time $t \leqslant \tau_1$, written on the CAT index over time interval $[\tau_1, \tau_2]$ as

$$F_{\mathrm{CAT}}(t, \tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} s(u)\,\mathrm{d}u + \kappa^{-1}(T(t) - s(t))(\mathrm{e}^{\kappa(\tau_2-t)} - \mathrm{e}^{\kappa(\tau_1-t)}) \\ + \kappa^{-1}\int_t^{\tau_2} \theta\sigma(u)(\mathrm{e}^{\kappa(\tau_2-u)} - 1)\,\mathrm{d}u \\ - \kappa^{-1}\int_t^{\tau_1} \theta\sigma(u)(\mathrm{e}^{\kappa(\tau_1-u)} - 1)\,\mathrm{d}u. \qquad (3.18)$$

Similarly to the energy future case, to derive the Q^θ dynamics of the CAT future price $\mathrm{d}F_{\mathrm{CAT}}(t, \tau_1, \tau_2)$, recall the Q^θ dynamics of $T(t)$ in (3.14) and apply Ito's lemma.[7] Thus, we have

$$\mathrm{d}F_{\mathrm{E}}(t, \tau_1, \tau_2) = \left(\frac{\partial F_{\mathrm{E}}}{\partial t} + \frac{1}{2}\frac{\partial^2 F_{\mathrm{E}}}{\partial T(t)^2}\right)\mathrm{d}t + \frac{\partial F_{\mathrm{E}}}{\partial T(t)}\,\mathrm{d}T(t). \qquad (3.19)$$

The price process $F_{\mathrm{E}}(t, \tau_1, \tau_2)$ is by construction a Q^θ martingale and the only coefficient for $T(t)$ in (3.18) is $\kappa^{-1}(\mathrm{e}^{\kappa(\tau_2-t)} - \mathrm{e}^{\kappa(\tau_1-t)})$; therefore,

$$\mathrm{d}F_{\mathrm{E}}(t, \tau_1, \tau_2) = \kappa^{-1}(\mathrm{e}^{\kappa(\tau_2-t)} - \mathrm{e}^{\kappa(\tau_1-t)})\sigma(t)\,\mathrm{d}W^\theta(t). \qquad (3.20)$$

Up to this point, we have already obtained the explicit dynamics of future prices under the P-equivalent martingale measure Q^θ for both energy futures and CAT futures. Next, we consider the dependence between the underlying energy spot price and average daily temperature. In a simple situation, this dependence could be modeled with a linear correlation between the sources of randomness of the spot price and temperature. Mathematically, under the real measure P for energy and temperature underlyings, we assume the energy spot price model in (3.1) and the temperature

[7] Denote $F_{\mathrm{CAT}}(t, \tau_1, \tau_2)$ as $F_{\mathrm{E}}(t, \tau_1, \tau_2)$.

model in (3.13) are driven by two linearly correlated standard Brownian motions, $W_{\mathrm{E}}(t)$ and $W_{\mathrm{W}}(t)$ respectively.[8] The time-dependent joint evolution of the Brownian motions is related to a correlation coefficient ρ as

$$\mathrm{d}W_{\mathrm{E}}(t)\,\mathrm{d}W_{\mathrm{W}}(t) = \rho\,\mathrm{d}t. \tag{3.21}$$

Note that this dependence relationship between the driving random variables is under a real measure P, but the derived future dynamics in (3.12) and (3.20) are related to Brownian motions under the pricing measure Q^{θ}. So, we need to figure out the dependence of $\mathrm{d}W_{\mathrm{E}}^{\theta}(t)$ and $\mathrm{d}W_{\mathrm{W}}^{\theta}(t)$ under the pricing Q^{θ} measure based on the Girsanov transformation. Recall that, through the Girsanov transformation, the Brownian motions for the energy market $\mathrm{d}W_{\mathrm{E}}^{\theta}(t)$ and weather market $\mathrm{d}W_{\mathrm{W}}^{\theta}(t)$ under the Q^{θ} measure are defined as

$$\begin{aligned}\mathrm{d}W_{\mathrm{E}}^{\theta}(t) &= \mathrm{d}W_{\mathrm{E}}(t) - \theta_{\mathrm{E}}\,\mathrm{d}t,\\ \mathrm{d}W_{\mathrm{W}}^{\theta}(t) &= \mathrm{d}W_{\mathrm{W}}(t) - \theta_{\mathrm{W}}\,\mathrm{d}t.\end{aligned}$$

Hence, the dependence between the two Brownian motions with respect to measure Q^{θ} is

$$\begin{aligned}\mathrm{d}W_{\mathrm{E}}^{\theta}(t)\,\mathrm{d}W_{\mathrm{W}}^{\theta}(t) &= (\mathrm{d}W_{\mathrm{E}}(t) - \theta_{\mathrm{E}}\,\mathrm{d}t)(\mathrm{d}W_{\mathrm{W}}(t) - \theta_{\mathrm{W}}\,\mathrm{d}t)\\ &= \mathrm{d}W_{\mathrm{E}}(t)\,\mathrm{d}W_{\mathrm{W}}(t) - \theta_{\mathrm{W}}\,\mathrm{d}W_{\mathrm{E}}(t)\,\mathrm{d}t - \theta_{\mathrm{E}}\,\mathrm{d}W_{\mathrm{W}}(t)\,\mathrm{d}t + \theta_{\mathrm{E}}\theta_{\mathrm{W}}\,\mathrm{d}t\,\mathrm{d}t\\ &= \mathrm{d}W_{\mathrm{E}}(t)\,\mathrm{d}W_{\mathrm{W}}(t) = \rho\,\mathrm{d}t.\end{aligned} \tag{3.22}$$

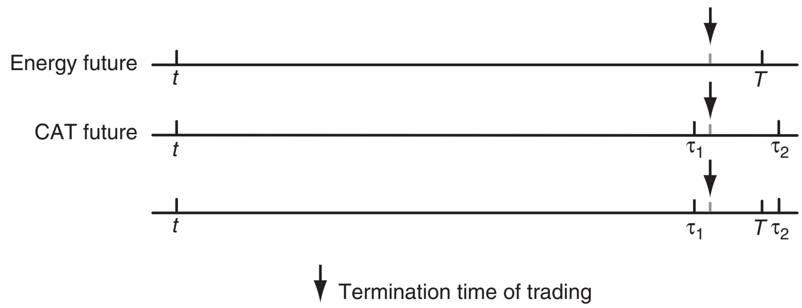
Then, from (3.12), (3.20) and (3.22), our combined Q^{θ} dynamics system for energy futures and CAT futures are

$$\left.\begin{aligned}\mathrm{d}F_{\mathrm{E}}(t,T) &= \sigma_{\mathrm{E}}\mathrm{e}^{-\kappa_{\mathrm{E}}(T-t)}\exp(\mu_X(T) + \tfrac{1}{2}\sigma_X^2(T))\,\mathrm{d}W_{\mathrm{E}}^{\theta}(t),\\ \mathrm{d}F_{\mathrm{W}}(t,\tau_1,\tau_2) &= \kappa_{\mathrm{W}}^{-1}(\mathrm{e}^{\kappa_{\mathrm{W}}(\tau_2-t)} - \mathrm{e}^{\kappa_{\mathrm{W}}(\tau_1-t)})\sigma_{\mathrm{W}}(t)\,\mathrm{d}W_{\mathrm{W}}^{\theta}(t),\\ \mathrm{d}W_{\mathrm{E}}^{\theta}(t)\,\mathrm{d}W_{\mathrm{W}}^{\theta}(t) &= \rho\,\mathrm{d}t.\end{aligned}\right\} \tag{3.23}$$

REMARK 3.1 Note that the futures dynamics in system (3.23) depends on different time scales. In particular, the dynamics of energy futures only depend on the current time t and the delivery time T, but the CAT futures dynamics depend on the current time t, the measurement starting time τ_1 and the ending time τ_2. First, based on the classical cross-hedge strategy, we know that the maturity of the futures contract

[8] The subscript E denotes "energy" and the subscript W denotes "weather".

FIGURE 2 Illustration of futures contract time scales.



should be close to the maturity of the hedge (see Hull 2005). Second, consider the futures price dynamics for both energy and temperature market trading at time t. The energy futures price $F_{\mathrm{E}}(t, T)$ with maturity (delivery) time $T > t \geqslant 0$ is theoretically the expected spot price at time T, whereas the CAT futures price $F_{\mathrm{W}}(t, \tau_1, \tau_2)$ is the theoretical expected cumulative temperature index during the measurement period $[\tau_1, \tau_2]$. It is natural to expect that the underlying variables for both futures are on the same time scale. Hence, the maturity time T for the energy futures should be located in the measurement time interval $[\tau_1, \tau_2]$ for CAT futures. In this paper, we set the termination of trading time for energy futures as equal to the termination time for temperature futures. Then, τ_1 and τ_2 can be selected as the available temperature futures with the shortest measurement time period, such that $\tau_1 < T < \tau_2$. The reason for this is that setting the termination of trading times the same will let the dynamic hedging strategy cover the entire trading period for both energy and temperature futures. Figure 2 illustrates this setting and the location of maturity time T in the interval $[\tau_1, \tau_2]$.

Finally, recall the dynamic hedge ratio β_t defined in (2.3). Based on the system of futures dynamics in (3.23), define

$$\left.\begin{aligned} c_1(t) &:= \sigma_{\mathrm{E}} \mathrm{e}^{-\kappa_{\mathrm{E}}(T-t)} \exp(\mu_X(T) + \tfrac{1}{2}\sigma_X^2(T)), \\ c_2(t) &:= \kappa_{\mathrm{W}}^{-1}(\mathrm{e}^{\kappa_{\mathrm{W}}(\tau_2 - t)} - \mathrm{e}^{\kappa_{\mathrm{W}}(\tau_1 - t)})\sigma_{\mathrm{W}}(t). \end{aligned}\right\} \tag{3.24}$$

Using the decomposition of a correlated Brownian motion (see Shreve 2004), the dependent relation $\mathrm{d}W_{\mathrm{E}}^{\theta}(t)\,\mathrm{d}W_{\mathrm{W}}^{\theta}(t) = \rho\,\mathrm{d}t$ is equivalent to

$$\mathrm{d}W_{\mathrm{E}}^{\theta}(t) = \rho\,\mathrm{d}W_{\mathrm{W}}^{\theta}(t) + \sqrt{1-\rho^2}\,\mathrm{d}\hat{W}_{\mathrm{W}}^{\theta}(t),$$

where $W_W^\theta(t)$ and $\hat{W}_W^\theta(t)$ are independent Brownian motions ($\Leftrightarrow$ $\mathrm{d}W_W^\theta(t)\,\mathrm{d}\hat{W}_W^\theta(t) = 0$). We have

$$\begin{aligned}\beta_t &= -\frac{\mathrm{d}F_E(t)}{\mathrm{d}F_W(t)} = -\frac{c_1(t)}{c_2(t)}\frac{\mathrm{d}W_E^\theta(t)}{\mathrm{d}W_W^\theta(t)} \\ &= -\frac{c_1(t)}{c_2(t)}\frac{\rho\,\mathrm{d}W_W^\theta(t) + \sqrt{1-\rho^2}\,\mathrm{d}\hat{W}_W^\theta(t)}{\mathrm{d}W_W^\theta(t)} \\ &= -\frac{c_1(t)}{c_2(t)}\left(\rho + \sqrt{1-\rho^2}\frac{\mathrm{d}\hat{W}_W^\theta(t)}{\mathrm{d}W_W^\theta(t)}\frac{\mathrm{d}W_W^\theta(t)}{\mathrm{d}W_W^\theta(t)}\right) \\ &= -\frac{c_1(t)}{c_2(t)}\left(\rho + \sqrt{1-\rho^2}\frac{0}{\mathrm{d}t}\right) = -\frac{c_1(t)}{c_2(t)}\rho, \end{aligned} \tag{3.25}$$

where $c_1(t)$ and $c_2(t)$ are time-dependent functions defined by (3.24).

4 NUMERICAL EXAMPLES

We now provide a numerical example for the dynamic hedge strategy proposed in Section 3.2. The first step for calculating the dynamic hedge ratio is the calibration of the two models in (3.1) and (3.13) from the energy market and temperature market, respectively. Then, with calibrated parameters (deterministic functions) in both models, we can get the dynamic hedge ratio β_t defined in (3.25).

First, in the energy market, without loss of generality, we choose to hedge the crude oil future using CAT futures.[9] Following the calibration method described in Schwartz (1997), the log-future prices $\ln F_E(t,T)$ need to be rewritten in the standard state-space form and then Kalman filtered to get the parameter set $\Theta_E = \{\kappa_E, \mu_E, \sigma_E, \theta_E\}$ and spot price series $S(t)$.

By taking the natural log of (3.10), for $t = 1,\dots,T$ and $i = 1,\dots,N$, we get the measurement equation

$$y_t = \mathrm{d}_t + Z_t x_t + \varepsilon_t, \tag{4.1}$$

where observations $y_t = \ln(F(t,T_i))$ with a dimension of $N \times 1$, and

$$d_t = \frac{1}{\kappa}\left(\theta\sigma + \kappa\left(\mu - \frac{\sigma^2}{2\kappa}\right)\right)(1 - \mathrm{e}^{-\kappa(T_i - t)}) + \frac{\sigma^2}{4\kappa}(1 - \mathrm{e}^{-2\kappa(T_i - t)})$$

with a dimension of $N \times 1$.[10] $Z_t = \mathrm{e}^{-\kappa(T_i - t)}$ with a dimension of $N \times 1$. The random noise ε_t is the $N \times 1$ vector of serially independent disturbances with $\mathbb{E}[\varepsilon_t] = 0$ and

[9] Crude oil is the world's most actively traded commodity, and the New York Mercantile Exchange (NYMEX) CME division's light, sweet crude oil futures contract is the world's most liquid forum for crude oil trading.

[10] The T represents the maximum date we considered in the future prices data set, and the N denotes the number of futures contracts taken into account in each trading day.

TABLE 2 Estimated parameters for the one-factor Schwartz model.

	μ	σ	κ	θ	ξ_1	ξ_2
Estimate	3.9187	0.0215	0.0025	0.2009	0.0003	0.0123

$\text{var}[\varepsilon_t] = H$. By discretizing (3.2), we can derive the transition equation

$$x_t = c_t + Q_t x_{t-1} + \eta_t, \tag{4.2}$$

where $c_t = (\mu - \sigma^2/2\kappa)(1 - e^{-\kappa\Delta t})$, $Q_t = e^{-\kappa\Delta t}$ and η_t is a serially independent normal random variable with $\mathbb{E}[\eta_t] = 0$ and $\text{Var}[\eta_t] = \sigma^2\Delta t$. Note that the state (latent) variable x_t and all coefficients in the transition equation are scalar.

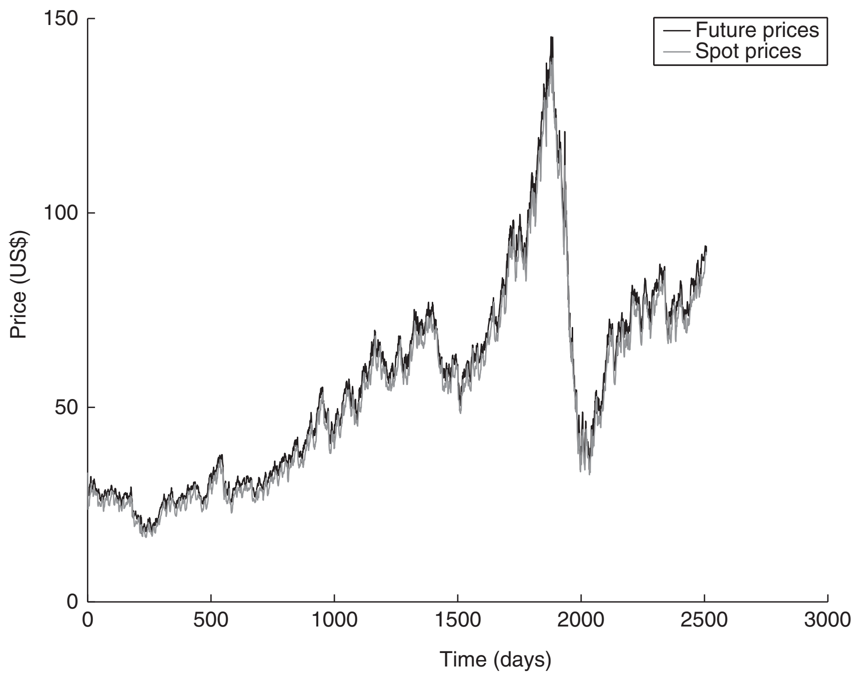
The state-space form in (4.1) and (4.2) is an enormously powerful tool that opens the way for handling the model with a latent factor using Kalman filtering. Following the Kalman filter algorithm in Harvey (1990) and Durbin and Koopman (2012), once the model has been put into the state-space form, the maximum likelihood estimation and Kalman filter are ready to be applied to get the parameters estimation and latent spot price series. The data used to calibrate the energy future consists of daily generic observations of West Texas Intermediate (WTI) light, sweet crude oil futures prices with delivery periods in the first two front months.[11] The WTI crude oil futures data used in calibration covers the CME exchange daily settlement prices ranging from January 2, 2001 to December 31, 2010, resulting in 2508 recorded for each futures contract set.[12] Since there is no exact delivery date for each contract, the CME contract specification instead defines a delivery period ranging from the first calendar day to the last calendar day of the delivery month. To calculate the time-to-maturity value $T_i - t$ in (4.1), we simply assume that the delivery date for each contract is the first calendar day in the delivery month.

Table 2 presents the estimation results for the energy model applied to the WTI crude oil futures price data. The last two parameters, ξ_1 and ξ_2, are the diagonal entries of matrix H in (4.1). Figure 3 on the facing page shows the daily estimated spot price (state variable) and the oil futures price for the contract closest to the delivery month ranging from January 2, 2001 to December 31, 2010.

[11] This data is obtained from Bloomberg financial services.

[12] This choice of data set is consistent with that in Swishchuk and Cui (2013), which includes ten years of temperature data from January 1, 2001 to December 31, 2010 in Calgary, Alberta, Canada.

FIGURE 3 Estimated daily spot price and futures price close to delivery.

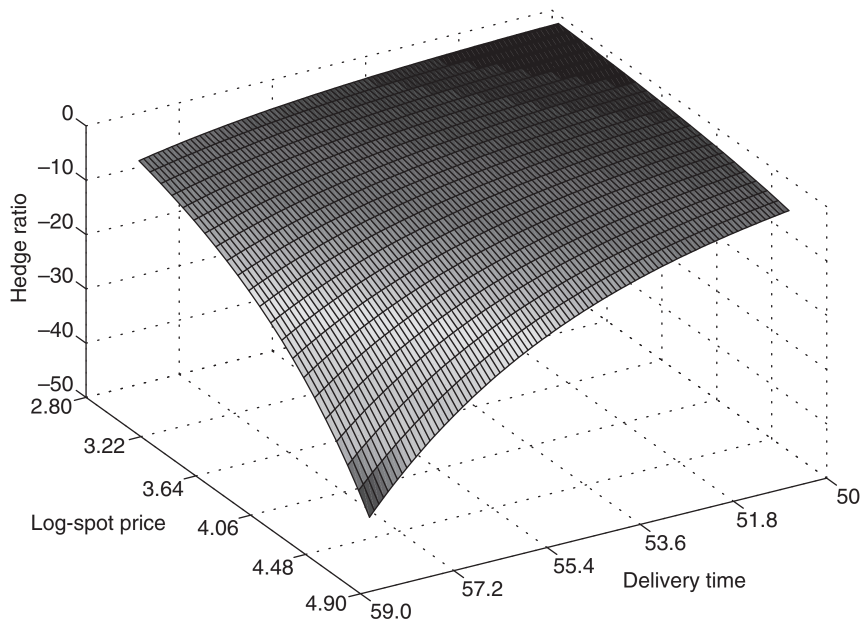


Next, for the temperature market, we follow the calibration procedure described in Swishchuk and Cui (2013) to get the parameter set $\Theta_W = \{\kappa_W, \sigma_W\}$.[13] Note that the temperature model (Ornstein–Uhlenbeck) in Swishchuk and Cui (2013) is driven by a general Lévy process instead of the Brownian motion in this paper, since the idea behind the calibration of the parameters in Swishchuk and Cui (2013) is to separate the seasonal, cyclical and volatility data step by step to finally get the random source. The estimates of the mean-reverting speed κ and volatility function σ will be the same under the model driven by Brownian motion in this paper. To illustrate this, we choose the estimated parameters in Calgary to be those under the temperature market to calculate the hedge ratio. Recalling the calibration results for Calgary in Swishchuk and Cui (2013), we get the parameter set $\Theta_W = \{\kappa_W, \sigma_W\}$ for Calgary as follows: $\kappa_W = -0.2411$, and the annual seasonal volatility

$$\sigma_W = 4.424 + 1.633\cos(0.0167t) + 0.1912\sin(0.0167t).$$

[13] Note that the dynamics of the CAT future price only depend on mean-reverting speed κ_W and volatility function σ_W under our temperature model setting.

FIGURE 4 Initial hedge ratio surface.



We now return to the dynamic hedge ratio β_t in (3.25). We have all of the necessary parameters in both the energy and weather markets, except for the correlation coefficient ρ between the two Brownian motions in these markets, since – from the model settings above – the only randomness of the underlyings in both the energy futures and the weather futures models is the Brownian motion. We can thus use the correlation between the filtered log-spot price and daily average temperature as a natural approximation to ρ. By taking all the average daily temperatures on the dates with futures prices available, and calculating the correlation coefficient between the log-spot prices and the average temperatures of these days over ten years (from January 2, 2001 to December 31, 2010), we have the correlation $\rho = 0.1058$. This correlation indicates a positive correlation between the log-spot price of crude oil and daily average temperature.

With the calibrated parameters in the energy and the temperature model, we can then calculate the dynamic hedge ratio β_t in (3.25) explicitly. In Figure 4, we plot the initial hedge ratio β_0 along the crude oil future delivery time (in days) and initial log-spot price dimensions. We find that, if we hold crude oil futures, initially we need to short some CAT futures in the portfolio, depending on the spot price of the crude oil and the time to delivery (trade termination) length. Basically, the number of temperature futures we need to hold will increase with increasing time to delivery and increasing spot price for the crude oil. Moreover, we can conclude that the same effect holds for other energy commodities, such as heating oil and gas, since they are usually positively correlated to movement in the crude oil market.

5 CONCLUSIONS AND FUTURE WORK

In terms of the applications of weather derivatives, by analyzing weather-related risks, we first concluded that the dominant risks for businesses are characterized as volume risks. Sticking to this point, we gave an example of using a temperature futures contract to statically hedge the descending sale risks. Looking at the issue of diversifying risks, we proposed a dynamic hedging strategy by using temperature futures for energy futures. The estimation of initial hedge ratios showed that the hedgers need more temperature futures when the delivery time of their energy futures and the current spot price increase.

The dynamic hedging strategy proposed in this paper can also be extended in several ways. For instance, the energy and temperature models in the system are both driven by a single source of randomness, Brownian motion. It would be more realistic and interesting to try the more general Lévy process or more sources of randomness (such as multifactor Schwartz's models or stochastic volatility models), if the dynamics of the energy and temperature futures could be obtained explicitly.

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